

$$\psi(x) = \frac{2A}{k_1 k_2} A e^{-k_2 x} = \frac{2A^2}{k_1 k_2} e^{-k_2 x}$$

$$P(x) = \frac{4E}{V^2} (V_1 V_2 - V E) |A|^2 e^{-k_2 x} = \frac{4E}{V^2} (V_1 V_2 - V E) |A|^2 e^{-k_2 x}$$

$$\Rightarrow P(x) = \frac{4E}{V^2} R_{ref} e^{-k_2 x}$$

③ Probabilité de réflexion en incidence

$$R = \frac{|R|^2}{|I|^2} = \frac{|B|^2}{|A|^2} = \left(\frac{A_2 - A_1}{A_1 + A_2} \right)^2 = \left(\frac{V_2 - V_1}{V_2 + V_1} \right)^2 < 1$$

④ Si $V_2 = 0 \Rightarrow k_2 = 0 \Rightarrow B = 0 \Rightarrow R = 0$
 Si $V_2 > V_1 \Rightarrow A_2 < A_1 \Rightarrow B \neq 0 \Rightarrow R > 0$

Exercice n° 7 d'octobre 2026-2027

Exercice n° 8: Barrière éparse en approximation "BKW."

$$(e) \Leftrightarrow \psi''(x) + \frac{2m}{\hbar^2} (E - V(x)) \psi(x) = 0$$

avec $V(x)$ d'allure proposée.

① $\psi(x) = f(x) e^{j\theta(x)}$

② $f'' \ll$ autres termes

a) $E_c = E - V(x) = \frac{p^2}{2m} \Rightarrow |p| = \sqrt{2m(E - V(x))}$

On injecte $\psi(x)$ dans l'ESIT:

$$\psi(x) = f(x) e^{j\theta(x)}$$

$$\psi'(x) = f'(x) e^{j\theta(x)} + j\theta'(x) f(x) e^{j\theta(x)}$$

$$\psi''(x) = f''(x) e^{j\theta(x)} + j f'(x) \theta'(x) e^{j\theta(x)} + (j\theta''(x) f(x) + j\theta'(x) f'(x)) e^{j\theta(x)} + (j\theta'(x))^2 f(x) e^{j\theta(x)}$$

$$d'au: (e) \Leftrightarrow \left[(f'' - \sigma'^2 f) + f(2\sigma\sigma' + \sigma''f) \right] + \frac{2m}{\hbar^2} (E - V(x)) f = 0$$

donc:
$$\begin{cases} f'' - \sigma'^2 f + \frac{2m}{\hbar^2} f = 0 \\ 2\sigma\sigma' + \sigma''f = 0 \end{cases} \Rightarrow \begin{cases} \sigma' = \frac{p(x)}{\hbar} \text{ (1) ans } \Leftrightarrow \text{à exclure} \\ f\sigma'' + 2\sigma\sigma' = 0 \text{ (2)} \end{cases}$$

une particule
x > en x0,
contenu des p(x)

b) (2) $\Rightarrow 2\sigma\sigma' = -f\sigma''$
 $\Rightarrow \frac{f'}{f} = -\frac{1}{2} \frac{\sigma''}{\sigma'} \stackrel{(1)}{\Rightarrow} \frac{f'}{f} = -\frac{1}{2} \frac{p'}{p}$

sait $\ln\left(\frac{f}{cte}\right) = +\ln p^{-1/2}$

$\Rightarrow f(x) = cte \times p^{-1/2}$

donc
$$\psi(x) = cte \left[\frac{2m(E - V(x))}{\hbar^2} \right]^{-1/4} e^{\frac{i}{\hbar} \int_0^x p(u) du}$$

② (H) $E < V(x)$ on recherche $\psi(x) = f(x) e^{-\sigma(x)}$. NB: evanescente

$$f\sigma'' + 2\sigma\sigma' = 0$$

$$\psi(x) = f(x) e^{-\sigma(x)}$$

$$\psi'(x) = f' e^{-\sigma} - \sigma' f e^{-\sigma}$$

$$\psi''(x) = f'' e^{-\sigma} - \sigma' f' e^{-\sigma} + (f\sigma'' - \sigma' f' + \sigma'^2 f) e^{-\sigma}$$

ES $\Rightarrow -\frac{\hbar^2}{2m} \psi''(x) + V(x)\psi(x) = E\psi(x) \Rightarrow \psi''(x) - \underbrace{\left(\frac{2m(V-E)}{\hbar^2} \right)}_{R^2} \psi(x) = 0$

donc: $\psi''(x) - R^2 \psi(x) = 0$

$$\Rightarrow f'' - \sigma' f' - \sigma'' f - \sigma' f' + \sigma'^2 f - R^2 f = 0$$

$$\Rightarrow -\underbrace{(f\sigma'' + 2\sigma\sigma')}_{\neq 0} + \underbrace{(f'' + f\sigma'^2)}_{\neq 0} - R^2 f = 0$$

$$\Rightarrow f\sigma'^2 = R^2 f \Rightarrow \sigma' = \sqrt{R^2} = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

$$\Rightarrow \sigma = \int \sqrt{\dots} dx$$

En représentant formellement l'équation sur f :

$$f \sigma'' + 2f' \sigma' = 0$$

$$\Rightarrow \frac{f'}{f} = -\frac{1}{2} \frac{\sigma''}{\sigma'} \Rightarrow \ln\left(\frac{f}{\sigma'}\right) = \ln C \sigma'^{-1/2}$$

$$\Rightarrow f = C \sigma' \sigma'^{-1/2}$$

donc $\varphi(x) = C e^{-\frac{2m(V-E)}{\hbar^2} x} - \int_0^x \sqrt{\frac{2m(V-E)}{\hbar^2}} dx$

donc: $T_{\text{WKB}} = \left| \frac{\varphi(x_2)}{\varphi(x_1)} \right|^2 = \frac{e^{-2 \int_0^{x_2} \sqrt{\dots} dx}}{e^{-2 \int_0^{x_1} \sqrt{\dots} dx}}$

$$\Rightarrow T_{\text{WKB}} = e^{-\frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m(V(x)-E)} dx}$$